

Type IIB flux vacua from the string worldsheet

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ABSTRACT: We study type IIB string compactifications in the presence of RR and NSNS fluxes using worldsheet techniques. Vertex operators corresponding to internal RR and NSNS fluxes are constructed in the hybrid formalism with manifest target space supersymmetry. In a simple class of examples, we compute various known physical phenomena such as warping and the generation of a superpotential for complex structure moduli. The results are in precise agreement with the supergravity literature.

KEYWORDS: Flux compactifications, Superstrings and Heterotic Strings, Conformal Field Models in String Theory.

Contents

1. Introduction	1
2. The hybrid formalism on Calabi-Yau 3-folds	3
2.1 Aspects of the string worldsheet	3
2.2 Four-dimensional sector	5
2.3 Internal Calabi-Yau sector	6
2.4 Physical states	8
3. Flux deformations	11
3.1 F_3 flux	12
3.2 H_3 flux	12
3.3 G_3 flux	15
3.4 F_1 and F_5 flux	15
4. Physical effects and applications	16
4.1 Conformally CY_3 backgrounds in supergravity	17
4.2 Conformally CY_3 backgrounds in string theory	18
4.3 Superpotential for moduli	20
5. Conclusion and future directions	21

1. Introduction

The study of string vacua is an important direction in string theory both from the point of view of constructing realistic phenomenology and in the context of ideas such as the ‘landscape’ of string vacua. To that end, it is important to understand how fluxes, especially Ramond-Ramond (RR) fluxes, affect the existence and modify the properties of string solutions. At low energies and at large volume, where α' corrections are under control, one can study string solutions using the techniques of supergravity. Already from the point of view of supergravity it is clear that RR vacua have some interesting features: Moduli are lifted and the space back-reacts, becoming warped. At string scale, α' corrections will modify the properties and existence of string solutions. For example, there is evidence for the existence of ‘non-geometric’ solutions of string theory that have no large volume limit (see [1] and references therein). To obtain a better understanding of such string scale solutions, one needs a string worldsheet description of RR flux vacua.

Studying flux backgrounds on the worldsheet has traditionally proven difficult. In the Ramond-Neveu-Schwarz (RNS) formulation, RR vertex operators carry non-zero picture

charge and have branch cuts in their operator product expansions, due to which, one cannot easily exponentiate them into the worldsheet action. The Green-Schwarz (GS) formalism circumvents some of these problems although the action cannot be covariantly quantized. Thus the GS approach is limited to a classical analysis and to backgrounds that admit light cone gauge. Alternative formalisms developed by Berkovits and collaborators have the nice feature of manifest space-time supersymmetry and admit a covariant quantization. These include the pure spinor formalism [2] and the hybrid formalism in $d = 2$ [3], $d = 4$ [4] and $d = 6$ [5, 6] which can be used to study RR backgrounds, as we do in this paper.

Previous studies of RR backgrounds have been undertaken in the context of $AdS_3 \times S^3$ [7], $AdS_2 \times S^2$ [8], the C-deformation [9], and non-commutative superspace [10, 11]. In all of these settings, the RR flux lies entirely in the uncompactified sector. RR fluxes in the compactified sector have been considered in [12–15]. For example, in [12] and later in [13, 14], the relation between internal fluxes, auxiliary fields, and the hybrid vertex operators was discussed.¹ In [13, 14] it was pointed out that the proposal for these flux vertex operators did not satisfy certain physical state conditions so it was not clear how to consistently construct flux deformations of the worldsheet action. In [15], a modified flux vertex operator was proposed that satisfied the physical state conditions. Open questions remained, however, concerning the precise physical meaning of the flux vertex operator and a systematic study of the associated space-time physics.

In this paper, we clarify and expand the presentation in [15]. We identify the vertex operators corresponding to both Ramond-Ramond and Neveu-Schwarz-Neveu-Schwarz (NSNS) fluxes in the hybrid formalism and resolve the issues alluded to in the previous paragraph. This is achieved by mapping the RNS flux vertex operators to the hybrid formalism via the auxiliary fields present in the $N = 2$ superspace formalism of [17]. This is an easier route than directly applying the RNS-hybrid map discussed in [5, 19–21]. Using our identification, we compute some of the physics associated with this flux deformation in the context of a simple, non-compact, conformally Calabi-Yau background. We study the effect of non-compactly supported fluxes in perturbation theory. This background also has a well-defined supergravity description with which we can compare our string calculations. Even in perturbation theory, we find many effects known from the supergravity literature. For example, we show how to derive warping (see also [15]) and a superpotential for complex structure moduli. All of these effects are in precise agreement with the existing supergravity literature. It should be stressed that we are considering non-compact Calabi-Yau spaces for two reasons. Firstly, it is not yet known how to deal with finite flux deformations, therefore we restrict ourselves to small fluxes. Secondly, in order to have compact solutions we need negative tension objects, which we do not include in the present paper.

The outline for this paper is as follows. In section 2 we give a schematic overview of the hybrid formalism, comparing and contrasting it with the RNS formulation. In section 3 we

¹After this work was completed, we were informed that in unpublished work, [16] obtained some related results in the context of type IIA on a Calabi-Yau 4-fold using the $d = 2$ Hybrid formalism of [3]. This included computing RR flux vertex operators, and computation of correlation functions which implied the presence of a superpotential.

identify the type IIB vertex operators for RR and NSNS flux deformations in the hybrid formalism. In section 4 we compute some simple physical effects and compare with known supergravity calculations. In section 5 we summarize our results and give an outlook on future work.

2. The hybrid formalism on Calabi-Yau 3-folds

In preparation for the discussion of flux deformations of Calabi-Yau compactifications of type IIB strings to 4 dimensions, we highlight some basic features of the $d = 4$ hybrid string worldsheet and contrast it with that of the RNS formulation (more details can be found in [4, 17, 19]).

2.1 Aspects of the string worldsheet

The (left-moving part of the) RNS formalism has a field content consisting of worldsheet bosons X^M and Majorana-Weyl fermions ψ^M (where $M = 0, 1 \dots, 9$) together with superconformal ghosts (b, c, β, γ) . The formulation has a manifest $N = (1, 1)$ worldsheet supersymmetry. This comes at the expense of manifest space-time supersymmetry: In this formalism, space-time supersymmetry requires the introduction of picture changing and complicated spin-fields with branch cuts in their operator products. This means that one cannot readily study RR backgrounds since the RR vertex operators are constructed from the spin-fields which cannot be easily exponentiated into the action.

The other major formulation of the string worldsheet theory is the Green-Schwarz formalism in which the fundamental field content consists of the worldsheet bosons X^M and Grassmann variables $\theta_L^\alpha, \theta_R^\alpha$. In this approach, target-space supersymmetry is realized linearly at the cost of manifest worldsheet supersymmetry. The Green-Schwarz formalism can classically describe fermionic and RR fields but is difficult to quantize due to the presence of second-class constraints inherent in its formulation. In order to quantize the worldsheet theory, one typically breaks Lorentz invariance (e.g. via light-cone gauge). The gauge choice complicates string amplitudes and restricts one to backgrounds that are compatible with the gauge choice. By contrast, important classes of flux backgrounds, including the ones we will consider, contain a space-time warp factor which is typically not consistent with the gauge choice. Thus, it is desirable to have a covariantly quantizable formulation in which space-time supersymmetry is manifest and RR fields can be treated naturally. For string compactifications preserving $SO(3, 1)$ Lorentz symmetry, the 4-dimensional hybrid formalism proposed by Berkovits [4] is a natural candidate.

Before discussing flux deformations, we review the hybrid formalism without flux as originally proposed in [4]. The 10-dimensional space-time manifold is taken to be of the form

$$\mathbb{R}^{3,1} \times M_6, \tag{2.1}$$

where M_6 is an internal manifold which preserves $N = 2$ space-time supersymmetry. In such a background the worldsheet theory factorizes. In general, the worldsheet theory will not factorize and the target space-time will not be a direct product as in (2.1) as discussed in section 4.

The hybrid formalism can be understood as a field redefinition of RNS. Take $M_6 = CY_3$ to be a Calabi-Yau 3-fold. Then the RNS worldsheet action splits as

$$S_{\text{RNS}} = S_{d=4} + S_{CY_3} + S_{\text{ghosts}}, \quad (2.2)$$

where $S_{d=4}$ describes the superconformal field theory parameterized by X^μ, ψ^μ , and S_{CY} describes an $N = (2, 2)$ superconformal field theory. We will parameterize the internal Calabi-Yau manifold by the complex coordinates $y^i, \bar{y}^{\bar{i}}$ and denote the corresponding complex Weyl fermions by $\psi_{L,R}^i, \bar{\psi}_{L,R}^{\bar{i}}$. The hybrid variables are obtained by performing a field redefinition on the space-time variables and ghost content together. That is, one replaces

$$(x^\mu, \psi^\mu, b, c, \beta, \gamma) \rightarrow (x^\mu, \theta_L^\alpha, \bar{\theta}_L^{\dot{\alpha}}, p_{L\alpha}, \bar{p}_{L\dot{\alpha}}, \rho_L), \quad (2.3)$$

and similarly for the right-moving sector, where $(\theta_{L,R}, \bar{\theta}_{L,R})$ are the usual $4d, N = 2$ Grassmann superspace coordinates, $(p_{L,R}, \bar{p}_{L,R})$ are their conjugate momenta, and (ρ_L, ρ_R) are two chiral ghost bosons. The spinor index indicates the space-time chirality. The (p, θ) variables are a (b, c) -type system with weights 1 and 0 respectively. The resulting action is a combination of GS in the 4-dimensional space and RNS in the internal space taking the form

$$S_{\text{hybrid}} = S_{GS,d=4} + S_{CY} + S_{\text{chiral}}. \quad (2.4)$$

Some general comments on this construction are:

- The space-time part of the target manifold is the standard $4d, N = 2$ superspace with coordinates, and 4-dimensional super-Poincaré symmetry is manifest. This allows a unified description of fermions and RR and NSNS fields.
- The action is free in a flat background and easily quantizable, lacking the second class constraints which complicate the quantization of the GS string.
- All the ghost content is now tied up in the definition the GS variables (p, θ) and the chiral bosons $\rho_{L,R}$ so there is no need to introduce additional ghosts in order to describe scattering processes.
- The field content is automatically GSO projected, and there are no branch cuts. This eliminates the difficult sum over spin structures essential for target space supersymmetry in RNS.
- The hybrid formalism has been subjected to many non-trivial tests and has passed them all. For example, the spectra of RNS and the hybrid formalisms match, as well as the tree [22] and 1-loop [23] amplitudes. It was shown that superconformal invariance at quantum level implies the correct four dimensional supergravity equations of motion for both heterotic and type IIB case [24, 25].

We will now discuss the space-time sector (and chiral bosons) and then turn to a discussion of the internal (Calabi-Yau) sector.

2.2 Four-dimensional sector

The 4-dimensional space-time superconformal field theory is described by a free field action for the GS like variables. The action is

$$S_{GS,d=4} = \int d^2z \left[\frac{1}{2} \partial_L X^\mu \partial_R X_\mu + p_{L\alpha} \partial_R \theta_L^\alpha + \bar{p}_{L\dot{\alpha}} \partial_R \bar{\theta}_L^{\dot{\alpha}} \right]. \quad (2.5)$$

The the resulting operator products are

$$\begin{aligned} X(z)^\mu X(0)^\nu &\sim -\eta^{\mu\nu} \ln |z|^2, \\ p_{L\alpha}(z) \theta_L^\beta(0) &\sim \frac{\delta_\alpha^\beta}{z}, & \bar{p}_{L\dot{\alpha}}(z) \bar{\theta}_L^{\dot{\beta}}(0) &\sim \frac{\delta_{\dot{\alpha}}^{\dot{\beta}}}{z}, \\ p_{R\alpha}(\bar{z}) \theta_R^\beta(0) &\sim \frac{\delta_\alpha^\beta}{\bar{z}}, & \bar{p}_{R\dot{\alpha}}(\bar{z}) \bar{\theta}_R^{\dot{\beta}}(0) &\sim \frac{\delta_{\dot{\alpha}}^{\dot{\beta}}}{\bar{z}}, \end{aligned} \quad (2.6)$$

while the chiral bosons have the operator products

$$\rho_L(z) \rho_L(0) \sim -\ln z, \quad \rho_R(\bar{z}) \rho_R(0) \sim -\ln \bar{z}. \quad (2.7)$$

The space-time supersymmetry charges are given by

$$\begin{aligned} Q_{L\alpha} &= \oint dz \left(p_{L\alpha} - \frac{i}{2} (\sigma^m \bar{\theta}_L)_\alpha \partial_L X_m - \frac{1}{8} (\bar{\theta}_L)^2 \partial_L \theta_{L\alpha} \right), \\ \bar{Q}_{L\dot{\alpha}} &= \oint dz \left(\bar{p}_{L\dot{\alpha}} - \frac{i}{2} (\theta_L \sigma^m)_{\dot{\alpha}} \partial_L X_m - \frac{1}{8} (\theta_L)^2 \partial_L \bar{\theta}_{L\dot{\alpha}} \right), \\ Q_{R\alpha} &= - \oint d\bar{z} \left(p_{R\alpha} - \frac{i}{2} (\sigma^m \bar{\theta}_R)_\alpha \partial_R X_m - \frac{1}{8} (\bar{\theta}_R)^2 \partial_R \theta_{R\alpha} \right), \\ \bar{Q}_{R\dot{\alpha}} &= - \oint d\bar{z} \left(\bar{p}_{R\dot{\alpha}} - \frac{i}{2} (\theta_R \sigma^m)_{\dot{\alpha}} \partial_R X_m - \frac{1}{8} (\theta_R)^2 \partial_R \bar{\theta}_{R\dot{\alpha}} \right). \end{aligned} \quad (2.8)$$

The set of Green-Schwarz-Siegel operators which commute with these charges are [26]

$$\begin{aligned} d_{L\alpha} &= p_{L\alpha} + \frac{i}{2} (\sigma^m \bar{\theta}_L)_\alpha \partial_L X_m - \frac{1}{4} (\bar{\theta}_L)^2 \partial_L \theta_{L\alpha} + \frac{1}{8} \theta_{L\alpha} \partial_L (\bar{\theta}_L)^2, \\ \bar{d}_{L\dot{\alpha}} &= \bar{p}_{L\dot{\alpha}} + \frac{i}{2} (\theta_L \sigma^m)_{\dot{\alpha}} \partial_L X_m - \frac{1}{4} (\theta_L)^2 \partial_L \bar{\theta}_{L\dot{\alpha}} + \frac{1}{8} \bar{\theta}_{L\dot{\alpha}} \partial_L (\theta_L)^2, \\ \Pi_L^\mu &= \partial_L X^\mu - \frac{i}{2} \sigma_{\alpha\dot{\alpha}}^\mu (\theta_L^\alpha \partial_L \bar{\theta}_L^{\dot{\alpha}} + \bar{\theta}_L^{\dot{\alpha}} \partial_L \theta_L^\alpha), \end{aligned} \quad (2.9)$$

together with their right-moving counterparts. The operator product expansion (2.6) implies that d_L and \bar{d}_L act on vertex operators as superspace covariant derivatives (c.f. equation (2.22)) and that these operators satisfy the algebra

$$\begin{aligned} d_{L\alpha}(z) \bar{d}_{L\dot{\alpha}}(0) &\sim i \frac{\Pi_{L\alpha\dot{\alpha}}}{z}, & d_{L\alpha}(z) d_{L\beta}(0) &\sim \text{regular}, & \bar{d}_{L\dot{\alpha}}(z) \bar{d}_{L\dot{\beta}}(0) &\sim \text{regular}, \\ d_{L\alpha}(z) \partial_L \theta_L^\beta(0) &\sim \frac{\delta_\alpha^\beta}{z^2}, & \bar{d}_{L\dot{\alpha}}(z) \partial_L \bar{\theta}_L^{\dot{\beta}}(0) &\sim \frac{\delta_{\dot{\alpha}}^{\dot{\beta}}}{z^2}, \\ d_{L\alpha}(z) \Pi_L^\mu(0) &\sim -i \frac{\sigma_{\alpha\dot{\alpha}}^\mu \partial_L \bar{\theta}_L^{\dot{\alpha}}(0)}{z}, & \bar{d}_{L\dot{\alpha}}(z) \Pi_L^\mu(0) &\sim -i \frac{\sigma_{\alpha\dot{\alpha}}^\mu \partial_L \theta_L^\alpha(0)}{z}, \\ \Pi_L^\mu(z) \Pi_L^\nu(0) &\sim -\frac{\eta^{\mu\nu}}{z^2}. \end{aligned} \quad (2.10)$$

There are analogous operator product expansions for the right movers.

The 4-dimensional space-time sector actually has a non-linear $N = (2, 2)$ superconformal symmetry with central charge $\hat{c} = -1$. The operators that generate it are given by

$$\begin{aligned}
 T_{L,d=4} &= -\frac{1}{2}\partial_L X^\mu \partial_L X_\mu - p_{L\alpha} \partial_L \theta_L^\alpha - \bar{p}_{L\dot{\alpha}} \partial_L \bar{\theta}_L^{\dot{\alpha}} - \frac{1}{2}\partial_L \rho_L \partial_L \rho_L, \\
 G_{L,d=4}^+ &= e^{\rho_L} (\bar{d}_L)^2, \quad G_{L,d=4}^- = e^{-\rho_L} (d_L)^2, \quad J_{L,d=4} = -\partial_L \rho_L,
 \end{aligned}
 \tag{2.11}$$

again, with analogous definitions for the right-moving side. The origin of this $N = (2, 2)$ supersymmetry lies in the relation between the parent RNS theory and hybrid reformulation: By requiring BRST and conformal invariance of the RNS string (with a gauged $N = (1, 1)$ supersymmetry) one can show that the RNS string possesses a twisted $N = (2, 2)$ supersymmetry [20]. This symmetry is preserved by the hybrid field redefinition and takes the form (2.11). We will use this $N = (2, 2)$ supersymmetry as a guiding principle when we discuss allowable flux deformations.

In the hybrid variables only 8 space-time supersymmetries are realized linearly. If the background preserves a larger number of supersymmetries (e.g. if $M_6 = T^6$) the remaining supersymmetries will be non-linearly realized. We will return to this when we discuss the F_5 flux in section 3.4.

We are now free to couple this $\hat{c} = -1$ superconformal field theory describing the space-time part of the target-space to a $\hat{c} = 3$ internal superconformal field theory. This gives a critical $\hat{c} = 2$, $N = 2$ string. To compute scattering amplitudes one could, in principle, add $\hat{c} = -2$ superconformal ghosts and perform BRST quantization. This was originally done in [21] and the results were shown in [20] to coincide with the $N = 1$ RNS string. However, it is easier to use the techniques described in [5] to embed this theory into an $N = 4$ topological theory after which one can compute amplitudes using the rules for topological strings. This is our approach in section 4.3.

2.3 Internal Calabi-Yau sector

The internal sector is described by a $\hat{c} = 3$, $N = (2, 2)$ superconformal field theory. The generators of the superconformal algebra will be denoted by $(\mathcal{T}, \mathcal{G}_L^+, \mathcal{G}_L^-, \mathcal{J}_L)$ where \mathcal{J}_L is the $U(1)$ R -symmetry generator. In section 3 the correspondence between chiral ring elements and harmonic forms will be used to identify worldsheet operators corresponding to fluxes on M_6 . In order to present a coherent picture, this identification will now be briefly reviewed for a Calabi-Yau target space. (Some nice reviews of $N = (2, 2)$ theories and chiral rings include [27, 28].)

An operator Φ is said to be a *chiral primary* operator if it is primary operator ($\mathcal{T}_n \cdot \Phi = \mathcal{G}_r^\pm \cdot \Phi = 0$ for $n, r > 0$) and, in addition, satisfies

$$(\mathcal{G}_L^+)_{-\frac{1}{2}} \cdot \Phi = 0.
 \tag{2.12}$$

Similarly, an anti-chiral operator satisfies the same conditions with $(\mathcal{G}_L^-)_{-\frac{1}{2}}$ replacing $(\mathcal{G}_L^+)_{-\frac{1}{2}}$. The set of chiral primary fields forms a ring under operator products, and chiral primary operators in a unitary representation saturate the bound $h \geq \frac{1}{2}q$ (where h is the

conformal weight and q the $U(1)_L$ charge). Analogous statements apply to the right-moving sector. The set of primaries that are chiral with respect to both the left- and right-moving algebra form the (c, c) -ring while elements of the (c, a) -ring are twisted-chiral primaries. Up to complex conjugation, these are the two possible chiral rings for a given $N = (2, 2)$ theory. In addition to chiral and twisted-chiral multiplets, there are also semi-chiral fields. These are chiral or anti-chiral only on the left-moving or right-moving side and are denoted $(c, -)$ or $(a, -)$. These will be important in the discussion of the NSNS flux deformations in section 3.

Elements of the chiral ring are classified by their $U(1)_L \times U(1)_R$ charges, and we have chosen the convention that chiral ring elements have positive charge while anti-chiral ring elements carry negative charge. By spectral flow, elements of the chiral ring can be mapped to Ramond ground states. Furthermore, for a Calabi-Yau target space at large volume, chiral ring elements map to harmonic forms on the CY. The ring will, in general, be α' corrected although it is expected to be robust under RG flow to the IR fixed point. Elements of the (c, a) -ring with $U(1)_L \times U(1)_R$ charges $(r, -s)$ can be mapped to elements of $H_{\bar{\partial}}^{r,s}(M_6)$. Similarly, elements of (c, c) -ring with charges (r, s) can be mapped to elements of $H_{\bar{\partial}}^{(r,0)}(M_6, \Lambda^{(0,s)}T)$ which are, in turn, associated with harmonic $(r, 3-s)$ -forms via contraction with the anti-holomorphic $(0, 3)$ -form $\bar{\Omega}$ of the Calabi-Yau. Chiral and twisted-chiral ring elements, as in RNS, correspond to complex structure and Kähler deformations respectively.

At large volume, the operators in the chiral rings have a representation in terms of the Weyl fermions which makes manifest the relation between chiral ring elements and harmonic forms. We review this representation now as it will be used in section 3 to construct flux deformations.

All operators in the internal CFT are classified by their $U(1)_L \times U(1)_R$ charges. We denote an operator with charges (p, q) by $\Omega_{p,q}$. Then a (c, a) -ring element $\Omega_{p,-q}$ with $U(1)_L \times U(1)_R$ charges $(p, -q)$ is (in the large radius limit):

$$\Omega_{p,-q} = \omega_{i_1 \dots i_p \bar{j}_1 \dots \bar{j}_q} \psi_L^{i_1} \dots \psi_L^{i_p} \bar{\psi}_R^{\bar{j}_1} \dots \bar{\psi}_R^{\bar{j}_q}, \quad (2.13)$$

where ω is a (p, q) -form. Physical state and chirality conditions imply $\omega \in H^{(p,q)}(\mathcal{M}_6)$. A (c, c) -ring element $\Omega_{p,q}$ corresponds to

$$\Omega_{p,q} = \omega_{i_1 \dots i_p \bar{j}_1 \dots \bar{j}_q} \psi_L^{i_1} \dots \psi_L^{i_p} \psi_{R\bar{j}_1} \dots \psi_{R\bar{j}_q}, \quad (2.14)$$

which, after applying physical state conditions, gives $\omega \in H^{(p,0)}(M_6, T^{(0,q)}M_6) \cong H^{(p,3-q)}(M_6)$.

To construct a critical string worldsheet theory we take the direct sum of the space-time and CY_3 superconformal algebras. The left-moving generators become

$$G_L^+ = e^{\rho_L} (\bar{d}_L)^2 + \mathcal{G}_L^+, \quad G_L^- = e^{-\rho_L} (d_L)^2 + \mathcal{G}_L^-, \quad J_L = -\partial_L \rho_L + \mathcal{J}_L, \quad (2.15)$$

while the right movers are²

$$G_R^+ = e^{\rho_R} (\bar{d}_R)^2 + \mathcal{G}_R^+, \quad G_R^- = e^{-\rho_R} (d_R)^2 + \mathcal{G}_R^-, \quad J_R = -\partial_R \rho_R + \mathcal{J}_R. \quad (2.16)$$

²This expression satisfies the type IIB GSO projection: The $4d$ space-time and internal CY chiralities are correlated. In type IIA they are anti-correlated.

2.4 Physical states

KK reduction of type IIB supergravity on a CY_3 . The massless spectrum of type IIB string theory on a Calabi-Yau 3-fold consists of a gravity multiplet, a tensor multiplet, and massless moduli (coming from the Kähler and complex structure deformations). The supergravity multiplet has bosonic components

$$G_{\mu\nu}, B_{\mu\nu}, \Phi, C_0, C_{\mu\nu}, C_{\mu ijk}. \quad (2.17)$$

Additionally there are Kähler moduli s^a with $a = 1, \dots, h^{(1,1)}$ and complex structure moduli ϕ^p with $p = 1, \dots, h^{(1,2)}$. The tensor fields are decomposed as

$$\begin{aligned} B_{MN} &= \sum_a r^a \omega_a^{(1,1)}, \\ C_{MN} &= \sum_a z^a \omega_a^{(1,1)}, \\ C_{MNPQ} &= \sum_a v^a \omega_a^{(1,1)} \wedge \omega_a^{(1,1)} + \sum_p A_\mu^p \omega_p^{(1,2)}. \end{aligned} \quad (2.18)$$

This results in 4 real scalars s^a, r^a, z^a, v^a for each $(1,1)$ -form grouped into a $4d, N = 2$ hypermultiplet. It is often convenient to write $\sigma^a = s^a + ir^a$ for a complexified Kähler modulus. There is also a $4d, N = 2$ vector multiplet (A_μ^p and ϕ^p) for each $(1,2)$ -form. We will now construct the corresponding hybrid vertex operators.

Physical states in string theory. Physical states in RNS are Virasoro primaries satisfying

$$L_n \cdot \mathcal{V} = G_r \cdot \mathcal{V} = 0 \quad (2.19)$$

for $n, r > 0$, modulo BRST exact states. The direct product structure (2.2) implies vertex operators are tensor products

$$\mathcal{V} = \mathcal{V}_{d=4} \otimes \mathcal{V}_{\text{int}}. \quad (2.20)$$

Elements of the gravity and tensor multiplets correspond to massless states with $\mathcal{V}_{\text{int}} = 1$. The massless moduli correspond to the case when \mathcal{V}_{int} is a chiral or twisted-chiral primary. The hybrid vertex operators are defined analogously except that $\mathcal{V}_{d=4}$ is constructed from 0-modes of $(x, \theta, \bar{\theta})$. Consequently, physical states are automatically $4d, N = 2$ superfields [17]. The multiplets are:

1. *Gravity and tensor multiplets:* $\mathcal{V} = U \otimes 1$.

These vertex operators are known as real primary fields. The physical state conditions require it to be an $N = 2$ Virasoro primary and real $\mathcal{V}^* = \mathcal{V}$. It can be shown using the algebra (2.6)–(2.10) and (2.15)–(2.16) that this implies the polarization conditions and equation of motion

$$\nabla_L^2 U = \bar{\nabla}_L^2 U = \nabla_R^2 U = \bar{\nabla}_R^2 U = \square_4 U = 0, \quad (2.21)$$

where

$$\nabla_{L\alpha} = \frac{\partial}{\partial \theta^\alpha} + i(\sigma^M \bar{\theta}_L)_\alpha \frac{\partial}{\partial X_M}, \quad \nabla_{R\alpha} = \frac{\partial}{\partial \theta^\alpha} + i(\sigma^M \bar{\theta}_R)_\alpha \frac{\partial}{\partial X_M} \quad (2.22)$$

are the superspace covariant derivatives. The equations (2.21) are the linearized 1-loop β -function equations.³ It can be shown that U is a superfield containing the $N = 2$ conformal supergravity multiplet and a tensor multiplet [17, 18]. The lowest component is given by

$$U = (\bar{\theta}_L \sigma^\mu \theta_L)(\bar{\theta}_R \sigma^\nu \theta_R)(h_{\mu\nu} + b_{\mu\nu} + \eta_{\mu\nu} A) + \dots, \quad (2.23)$$

where $h_{\mu\nu}$ is the symmetric traceless metric, $b_{\mu\nu}$ the B -field and A is related to the trace of the metric.⁴ The omitted fields are of higher dimension and will not be relevant for the discussions in this paper. The integrated vertex operator is

$$\int d^2z (G_L^+)_{-\frac{1}{2}} (G_L^-)_{-\frac{1}{2}} (G_R^+)_{-\frac{1}{2}} (G_R^-)_{-\frac{1}{2}} U. \quad (2.24)$$

Explicitly this becomes

$$\int d^2z \{ \mathcal{O}_L \cdot \mathcal{O}_R \cdot U \}$$

for

$$\begin{aligned} \mathcal{O}_L = & d_{\alpha L} \bar{\nabla}_L^2 \nabla_L^\alpha - \bar{d}_{\dot{\alpha} L} \nabla_{\dot{\alpha} L} \bar{\nabla}_L^2 + \partial \theta_L^\alpha \nabla_{\alpha L} - \partial \bar{\theta}_L^{\dot{\alpha}} \bar{\nabla}_{\dot{\alpha} L} - i \Pi_{\alpha \dot{\alpha}} [\nabla_L^\alpha, \bar{\nabla}_L^{\dot{\alpha}}] \\ & + \mathcal{G}_L^+ \cdot \mathcal{G}_L^- + e^{\rho L} \bar{d}_{L \dot{\alpha}} \bar{\nabla}_L^{\dot{\alpha}} \mathcal{G}_L^- + e^{-\rho L} d_{L \alpha} \nabla_L^\alpha \mathcal{G}_L^+ \end{aligned}$$

(and the analogous expression for \mathcal{O}_R), where all operators act on U .

Recall that the \mathcal{G} s are the internal CY generators; in general we can allow U to be a function of the coordinates of the internal manifold. However, this would imply that the last physical state condition in (2.21) is not satisfied, since U will have double poles with \mathcal{T} . We will return to this in section 4 when we consider flux perturbation theory and the β -function. There we will see that this violation is of second order in the flux. This, in turn, implies that U breaks conformal invariance at 1-loop in α' . We will ignore this issue for now and return to it in section 4. We conclude by noting that it is not possible for U to depend on the fermions of the internal theory since this breaks conformal invariance already at classical level.

2. *Vector multiplet:* $\mathcal{V}_{CC} = \mathcal{V}_{d=4} \otimes \mathcal{V}_{\text{int}}$ with $\mathcal{V}_{\text{int}} \in (c, c)$ -ring.

In type IIB string theory, vector multiplets and chiral ring elements correspond to complex structure deformations. The physical state conditions impose that the vertex operator is a massless Virasoro primary (2.19) and satisfies a chirality constraint:

$$(G_L^+)_{-\frac{1}{2}} \cdot \mathcal{V}_{CC} = 0, \quad (G_R^+)_{-\frac{1}{2}} \cdot \mathcal{V}_{CC} = 0. \quad (2.25)$$

³To be more precise, we could interpret this as “super- β -function equations” [24, 25] since they come from the requirement that the superconformal generators still satisfy the superconformal algebra after the action is deformed by this vertex operator.

⁴In fact, the scalar field A sits in the tensor multiplet. Only after fixing space time conformal symmetry is this component related to the trace of the metric.

This implies that $\mathcal{V}_{\text{int}} \in (c, c)$ -ring and $\mathcal{V}_{d=4}$ is a $4d$, $N = 2$ chiral superfield which has the component expansion

$$\mathcal{V}_{d=4} = \phi + \theta_L^2 D_{--} + \theta_R^2 D_{++} + \theta_L \theta_R D_{+-} + \theta_L \sigma^{\mu\nu} \theta_R F_{\mu\nu} + \theta_L^2 \theta_R^2 \square \bar{\phi} + \text{fermions}, \quad (2.26)$$

where ϕ is the complex structure modulus. The D_{ij} are auxiliary fields with i, j $SU(2)$ R -symmetry indices, while $F_{\mu\nu}$ is the field strength for the vector field. This is the field content of the $N = 2$ vector multiplet. The physical state conditions (2.19) put (2.26) on-shell

$$\nabla_L^2 \mathcal{V}_{d=4}(x, \theta_L, \theta_R) = \nabla_R^2 \mathcal{V}_{d=4}(x, \bar{\theta}_L, \bar{\theta}_R) = 0 \quad (2.27)$$

which removes the auxiliary fields and imposes the equations of motion on the physical field content. The integrated vertex operator is of the form

$$\int d^2 z G_{L, -\frac{1}{2}}^- G_{R, -\frac{1}{2}}^- \mathcal{V}_{\text{CC}} + \int d^2 z G_{L, -\frac{1}{2}}^+ G_{R, -\frac{1}{2}}^+ \bar{\mathcal{V}}_{\text{CC}}, \quad (2.28)$$

where the complex conjugate is included for unitarity.

3. *Hypermultiplets*: $\mathcal{V} = \mathcal{V}_{d=4} \otimes \mathcal{V}_{\text{int}}$ with $\mathcal{V}_{\text{int}} \in (c, a)$ -ring.

The hypermultiplets in type IIB correspond to Kähler deformations and are twisted-chiral ring elements. The vertex operator is a massless Virasoro primary satisfying (2.19) and a twisted chirality constraint:

$$(G_L^+)_{-\frac{1}{2}} \cdot \mathcal{V}_{CA} = 0, \quad (G_R^-)_{-\frac{1}{2}} \cdot \mathcal{V}_{CA} = 0. \quad (2.29)$$

These conditions imply that $\mathcal{V}_{\text{int}} \in (c, a)$ -ring. The corresponding $4d$ superfield has a theta expansion

$$\mathcal{V}_{d=4} = l_{++} + \theta_L^2 y + \bar{\theta}_R^2 \bar{y} + \theta_L \sigma^\mu \bar{\theta}_R (\partial_\mu l_{+-} + (*H)_\mu) + \theta_L^2 \bar{\theta}_R^2 \square l_{--} + \text{fermions}, \quad (2.30)$$

where l_{ij} are scalars in the tensor representation of $SU(2)_R$, y, \bar{y} are auxiliary fields and $H_{\mu\nu\lambda}$ is a tensor field strength. This is the field content of the $N = 2$ hypermultiplet. The primary conditions

$$\nabla_L^2 \mathcal{V}_{d=4}(x, \theta_L, \theta_R) = \bar{\nabla}_R^2 \mathcal{V}_{d=4}(x, \bar{\theta}_L, \bar{\theta}_R) = 0. \quad (2.31)$$

put the field content on-shell. In components these equations fix the auxiliary fields and imply the equations of motion and polarization conditions for the remaining component fields. The integrated vertex operator is of the form

$$\int d^2 z (G_L^-)_{-\frac{1}{2}} (G_R^+)_{-\frac{1}{2}} \mathcal{V}_{CA} + \int d^2 z (G_L^+)_{-\frac{1}{2}} (G_R^-)_{-\frac{1}{2}} \bar{\mathcal{V}}_{CA}. \quad (2.32)$$

With the (space-time) \otimes (internal) structure of the vertex operators understood, we will drop the cumbersome tensor product notation henceforth.

3. Flux deformations

In the previous section, we listed the physical states of string theory as hybrid vertex operators. As in RNS, these vertex operators can also be used to construct deformations of theory. For example, one can construct integrated vertex operators corresponding to complex structure and Kähler deformations in the obvious way as

$$\int d^2z (G_L^-)_{-\frac{1}{2}} (G_R^-)_{-\frac{1}{2}} \cdot (\phi_p \Omega_{(1,1)}^p) \quad \text{and} \quad \int d^2z (G_L^-)_{-\frac{1}{2}} (G_R^+)_{-\frac{1}{2}} \cdot ((l_{++})_k \Omega_{1,-1}^k), \quad (3.1)$$

where $p = 1, \dots, h^{(1,2)}$ and $k = 1, \dots, h^{(1,1)}$ label a basis of (c, c) and (c, a) operators of the internal CFT respectively.

In addition to the massless moduli, there are also RR and NSNS fluxes. The fluxes of most interest to us are the 3-form fluxes on the internal space. In supergravity these may be decomposed as

$$\begin{aligned} F_{MNP} &= \sum F_{(1,2)}^p \omega_p^{(1,2)} + \text{c.c.} \\ H_{MNP} &= \sum_p H_{(1,2)}^p \omega_p^{(1,2)} + \text{c.c.} \end{aligned} \quad (3.2)$$

The vertex operators for these fluxes are well-known in RNS, and in particular, the RR vertex operators have branch cuts in their operator products. The remarkable property of the hybrid is that even though the internal variables are represented in the RNS formalism, the manifest space-time supersymmetry removes the branch cuts complicating the space-time part of that formalism. The resulting GSO projected RR flux vertex operators may consistently be integrated into the action. In this section we will construct the flux vertex operators. Since it is not yet known how to deal with finite deformations of such vertex operators, they will be considered as small perturbations of the action.

Our starting point will be the RNS formalism. The space-time supercharges are given in the $-\frac{1}{2}$ -picture by

$$\begin{aligned} Q_{L\alpha} &= \oint dz e^{-\frac{1}{2}\phi_L} \Sigma_{L\alpha} e^{-i\sqrt{3}H_L/2}, & \bar{Q}_{L\dot{\alpha}} &= \oint dz e^{-\frac{1}{2}\phi_L} \bar{\Sigma}_{L\dot{\alpha}} e^{i\sqrt{3}H_L/2}, \\ Q_{R\alpha} &= \oint d\bar{z} e^{-\frac{1}{2}\phi_R} \Sigma_{R\alpha} e^{-i\sqrt{3}H_R/2}, & \bar{Q}_{R\dot{\alpha}} &= \oint d\bar{z} e^{-\frac{1}{2}\phi_R} \bar{\Sigma}_{R\dot{\alpha}} e^{i\sqrt{3}H_R/2}, \end{aligned} \quad (3.3)$$

where Σ_L, Σ_R are the left- and right-moving spin fields and $e^{\pm i\sqrt{3}H_{L,R}/2}$ is the operator implementing spectral flow by half a unit. The GSO projection has been applied, that is, all four Q 's above have eigenvalues -1 with respect to $e^{i\pi F}$.

Now consider the RNS vertex operator for the complex structure modulus ϕ^p , which is an element of the (c, c) -ring. That is,

$$k^2 \phi^p = e^{-\phi_L - \phi_R} e^{ikX} \Omega_{(1,1)}^p, \quad (3.4)$$

where $\Omega_{(1,1)}^p = \omega_i^{\bar{j}}(X) \psi_L^i \psi_{R,\bar{j}}$ and $\omega \in H^1(M, TM)$. We will now use equation (3.4) and the $N = 2$ space-time supersymmetry algebra of the supercharges (3.3) to identify the vertex operators with the auxiliary fields D_{ij} and hence fluxes in the hybrid. We follow the techniques developed in [29, 30]. Some recent analysis along these lines can be found in [14, 31].

3.1 F_3 flux

The RR vertex operator for F_3 flux on the internal space is constructed as a spinor bilinear in the RNS spin-fields. One can check that the correct vertex operator is

$$e^{-\frac{1}{2}\phi_L - \frac{1}{2}\phi_R} \epsilon^{\alpha\beta} \Sigma_{L\alpha} \Sigma_{R\beta} \Psi_{RR}^p, \quad (3.5)$$

where α, β are $4d$ spinor indices and Ψ_{RR}^p is the RR ground state in the internal CFT corresponding to the $(1, 2)$ -cohomology representative. From the space-time supersymmetry algebra we have $k^2 \phi^p = \epsilon^{\dot{\alpha}\dot{\beta}} \{ \bar{Q}_{L,\dot{\alpha}}, [\bar{Q}_{R,\dot{\beta}}, D_{+-}^p] \}$. Following [14, 29–31] and using (3.3) and (3.4) we can identify

$$D_{+-}^p \leftrightarrow ig_s F_{(1,2)}^p \leftrightarrow g_s e^{-\frac{1}{2}\phi_L - \frac{1}{2}\phi_R} \epsilon^{\alpha\beta} \Sigma_{L,\alpha} \Sigma_{R,\beta} \Psi_{RR}^p. \quad (3.6)$$

This identification agrees with the analysis in [14]. Using the component expansion (2.26) and the correspondence (3.6) we can identify the hybrid vertex operator corresponding to $F_{(1,2)}$ flux as

$$\mathcal{V}_{F_{(1,2)}} = ig_s \theta_L \theta_R F_{(1,2)}^p \Omega_{(1,1)}^p. \quad (3.7)$$

This satisfies the physical state conditions (2.19) and is a chiral ring element in the full $d = 10$ hybrid theory. It is now possible to see, schematically at least, how the RNS to hybrid map is working:

- The spin fields and ghosts map as $e^{-\phi_L - \phi_R} \epsilon^{\alpha\beta} \Sigma_{L\alpha} \Sigma_{R\beta} \leftrightarrow \theta_L \theta_R$.
- The RR ground state is transformed by spectral flow to the NSNS sector: $\Psi_{RR}^p \leftrightarrow \Omega_{(1,1)}^p$.

As the $\theta_{L,R}$ variables have worldsheet spin 0, they have no branch cuts in their operator products. Using the form of the integrated vertex operator (2.28) we can write down the explicit deformation of the action:

$$\begin{aligned} \delta S_{F_3} = ig_s F_p^{(1,2)} \int d^2 z \left\{ e^{-\rho_L - \rho_R} d_L d_R \Omega_{(1,1)}^p + (\theta_L \theta_R) (\mathcal{G}_L^- \mathcal{G}_R^- \cdot \Omega_{(1,1)}^p) + \right. \\ \left. + e^{-\rho_L} (d_L \theta_R) (\mathcal{G}_{R,-\frac{1}{2}}^- \Omega_{(1,1)}^p) + e^{-\rho_R} (d_R \theta_L) (\mathcal{G}_{L,-\frac{1}{2}}^- \Omega_{(1,1)}^p) \right\} + \text{c.c.} \end{aligned} \quad (3.8)$$

The complex conjugate corresponds to $F_{(2,1)}^p$ fluxes in the obvious way and is required for unitarity.

3.2 H_3 flux

In the RNS σ -model describing a Calabi-Yau background, a deformation of the worldsheet action by a certain H_3 flux corresponds to

$$\begin{aligned} \delta S = \frac{1}{4\pi} \int d^2 z \left\{ (\delta G_{i\bar{j}}(X) + \delta B_{i\bar{j}}(X)) \partial_L X^i \partial_R X^{\bar{j}} + \left(\delta \Gamma_{\bar{i}j k}(X) + \frac{1}{2} \delta H_{\bar{i}j k}(X) \right) \bar{\psi}_L^{\bar{i}} \bar{\psi}_L^{\bar{j}} \partial_R X^k \right. \\ \left. + \left(\delta \Gamma_{\bar{i}j k}(X) - \frac{1}{2} \delta H_{\bar{i}j k}(X) \right) \bar{\psi}_R^{\bar{i}} \bar{\psi}_R^{\bar{j}} \partial_L X^k + \text{c.c.} + \dots \right\}, \end{aligned} \quad (3.9)$$

where all quantities are small deformations of the $N = (2, 2)$ Calabi-Yau background: δB is the B -field deformation and δG is the associated metric deformation required to preserve supersymmetry. $\delta\Gamma$ is the deformation of the Levi-Civita connection due to δG while δH is the field strength corresponding to δB . The omitted terms are the 4-fermion terms which will not be relevant for the discussion in this section. All of these terms are required to preserve worldsheet supersymmetry. For notational convenience we drop the δ henceforth, keeping in mind that these operators are to be regarded as small deformations.

We now turn to the construction of the analogue of the RNS deformation (3.9) in the hybrid formalism. First, we construct the vertex operators for the torsional connection $\Gamma \pm \frac{1}{2}H$. Then, we identify the vertex operator for the complexified metric and point out that only the total vertex operator is required to be physical. This resolves the issue discussed in [13–15] of how to consistently turn on fluxes on the internal space.

The $N = 2$ supersymmetry algebra implies that the D_{++} auxiliary component field satisfies

$$\epsilon^{\dot{\alpha}\dot{\beta}}\{\bar{Q}_{R\dot{\alpha}}, [\bar{Q}_{R\dot{\beta}}, D_{++}^p]\} = k^2\phi^p. \tag{3.10}$$

Using the same procedure as that above, we find the corresponding RNS vertex operator

$$D_{++}^p \leftrightarrow \omega_{i\bar{j}\bar{k}}^p \partial_L X^i \bar{\psi}_R^{\bar{j}} \bar{\psi}_R^{\bar{k}}. \tag{3.11}$$

Here, $\omega^p \in H^{1,2}(M)$ and we have applied picture changing so that the vertex operator is in the $(0, 0)$ -picture. From the deformation (3.9), we read off that the term $\bar{\psi}_R \bar{\psi}_R \partial_L X$ couples to a combination of the Christoffel connection Γ_{NP}^M and the B -field. Thus, we identify⁵

$$D_{++}^p \leftrightarrow i\Gamma_{(1,2)}^p + \frac{i}{2}H_{(1,2)}^p. \tag{3.12}$$

Similarly, by interchanging left-movers and right-movers, one identifies⁶

$$D_{--}^p \leftrightarrow i\Gamma_{(1,2)}^p - \frac{i}{2}H_{(1,2)}^p. \tag{3.13}$$

With this, the hybrid vertex operator corresponding to the torsional connection in (3.9) becomes

$$\mathcal{V}_{H_3} \sim i \left(\Gamma_{(1,2)}^p - \frac{1}{2}H_{(1,2)}^p \right) \theta_L^2 \Omega_{(1,1)}^p + i \left(\Gamma_{(1,2)}^p + \frac{1}{2}H_{(1,2)}^p \right) \theta_R^2 \Omega_{(1,1)}^p. \tag{3.14}$$

As it stands, this vertex operator is not a Virasoro primary since it has a non-zero double-pole with $G_{L,R}^-$. Equivalently, this vertex operator does not satisfy the physical state conditions. This implies that we cannot turn on this vertex operator since it corresponds to auxiliary fields which are forced to vanish by the equations of motion. This is the puzzle raised in [13, 14], a solution to which was proposed in [15]: One should add other

⁵We deduce the presence of the i either through the corresponding vertex operator for the metric or the B -field.

⁶This identification differs from the results of the analysis in [14] in which it was claimed that $D_{\pm\pm}^p \leftrightarrow T_{(1,2)}^p \pm H_{(1,2)}^p$ with T is the (metric) torsion tensor.

terms violating the physical state conditions in such a way that the sum does not, and consequently, is physical. From the explicit formula for the deformation (3.9) above, we can now give a physical interpretation to these pieces: The vertex operator (3.14) is missing the metric and B -field.

Motivated by [15] we are led to propose the following vertex operator for the NSNS flux:

$$\mathcal{V}_{H_3} = i \left(\Gamma_{(1,2)}^p - \frac{1}{2} H_{(1,2)}^p \right) \theta_L^2 \Omega_{(1,1)}^p + i \left(\Gamma_{(1,2)}^p + \frac{1}{2} H_{(1,2)}^p \right) \theta_R^2 \Omega_{(1,1)}^p + e^{-\rho_L} E_{(2,1)} + e^{-\rho_R} E_{(1,2)}, \quad (3.15)$$

where the operators $E_{(i,j)}$ have $U(1)$ charges (i, j) . Let us now interpret the last two terms. We require this operator to be physical in order to preserve $N = (2, 2)$ worldsheet supersymmetry. The physical state conditions imply for the left-movers that

$$\mathcal{G}_{L, \frac{1}{2}}^- \cdot E_{(2,1)} = i \left(\Gamma_{(1,2)}^p - \frac{1}{2} H_{(1,2)}^p \right) \Omega_{(1,1)}^p. \quad (3.16)$$

The operator $E_{(2,1)}$ is, therefore, not in the chiral ring. To interpret it we can go to the large radius limit, where

$$E_{(2,1)} = E_{ij}{}^{\bar{k}} \psi_L^i \psi_L^j \psi_{R, \bar{k}} = \Omega_L^+ (E_{ij}{}^{\bar{k}} \bar{\psi}_L^{\bar{i}} \bar{\psi}_L^{\bar{j}} \psi_R^{\bar{k}}), \quad (3.17)$$

and $\Omega_L^+ = \Omega_{ijk} \psi_L^i \psi_L^j \psi_L^k$ is the operator corresponding to the holomorphic 3-form. Then,

$$\mathcal{G}_{L, \frac{1}{2}}^- \cdot E_{(2,1)} = \Omega_L^+ (-i \bar{\partial}_{\bar{i}} E_{\bar{j}k} \bar{\psi}_L^{\bar{i}} \bar{\psi}_L^{\bar{j}} \psi_R^k). \quad (3.18)$$

Noting that

$$\left(\Gamma_{\bar{i}j k}^p - \frac{1}{2} H_{\bar{i}j k}^p \right) = -i (\bar{\partial}_{\bar{i}} G_{\bar{j}k} + \bar{\partial}_{\bar{j}} B_{\bar{i}k}), \quad (3.19)$$

and plugging this and (3.18) into (3.16), we find that⁷

$$E_{2,1} = -i \Omega_L^+ (G_{\bar{i}j} + B_{\bar{i}j}) \bar{\psi}_L^{\bar{i}} \psi_R^j. \quad (3.20)$$

Thus, this new term corresponds to the complexified metric. The right-movers follow analogously. By construction, the full vertex operator (3.15) satisfies the physical state conditions. It may be used to deform the action by the formula for the integrated operator (2.28).

We close this section with the observation that the operators $B_{(2,1)}$ and $B_{(1,2)}$ are semi-chiral fields. This implies that the hybrid description of NSNS flux backgrounds is intimately related to the torsional σ -models discussed in [32, 33] and references therein. Direct investigation of the interplay between generalized geometry and target space supersymmetry should be possible using this observation.

⁷The operator $e^{-\rho_L} \Omega_L^+$ is an $SU(2)_R$ rotation which arises when one embeds the critical $N = 2$ string as a topological $N = 4$ theory. See for example [5].

3.3 G_3 flux

Using the results of sections 3.1 and 3.2 it is now straightforward to write down the vertex operator corresponding to $G_3 = F_3 - \tau H_3$, which is the relevant flux for conformally Calabi-Yau compactifications. We will set the Ramond-Ramond scalar $C_0 = 0$ here for simplicity,

It will be convenient to perform a field redefinition on the vector multiplet components (2.26) so that

$$\mathcal{V}_{d=4} = \phi^p + (\theta^-)^2 X_{--}^p + (\theta^+)^2 X_{++}^p + \theta^- \theta^+ X_{+-}^p + \dots, \quad (3.21)$$

where $\theta^\pm = \theta_L \pm i\theta_R$, and the new fields are

$$\begin{aligned} 4X_{++}^p &= D_{++}^p - D_{--}^p - iD_{+-}^p, \\ 4X_{--}^p &= D_{++}^p - D_{--}^p + iD_{+-}^p, \\ 2X_{+-}^p &= D_{++}^p + D_{--}^p. \end{aligned} \quad (3.22)$$

The identifications of the vector multiplet auxiliary fields (3.6), (3.12) and (3.13) give

$$4X_{++} = g_s(F_{(1,2)}^p + ig_s^{-1}H_{(1,2)}^p) = g_s\overline{G}_{(2,1)}, \quad 4X_{--} = g_s(F_{(1,2)}^p - ig_s^{-1}H_{(1,2)}^p) = g_sG_{(1,2)}. \quad (3.23)$$

Suppose we want to turn on $G_{(2,1)}$ flux and keep $G_{(1,2)} = 0$. This corresponds to the vertex operator

$$\mathcal{V}_{G_{(2,1)}} = (\bar{\theta}_L - i\bar{\theta}_R)^2 g_s G_{(2,1)}^p \Omega_{(-1,-1)}^p, \quad \mathcal{V}_{\overline{G}_{(2,1)}} = (\theta_L + i\theta_R)^2 g_s \overline{G}_{(2,1)}^p \Omega_{(1,1)}^p, \quad (3.24)$$

where $G_{(2,1)}^p$ is the amount of G_3 flux through the homology cycle labelled by $p = 1, \dots, h^{(2,1)}$. The integrated vertex operator is easily constructed using the rule (2.28).

Let us pause for a moment to make some comments. Firstly, giving an expectation value to an auxiliary field multiplying θ^2 breaks the corresponding supersymmetry. For example the vertex operator $\mathcal{V} \propto (\theta_L + i\theta_R)^2$ breaks the corresponding space-time supersymmetry generated by $(Q_L + iQ_R)$. Secondly, we see from the explicit form (3.24) of the $G_{(2,1)}$ vertex operator that turning on (2,1)-flux is compatible with the $N = 1$ supersymmetry preserved by a D3-brane, as expected. Conversely, $G_{(1,2)}$ corresponds to anti-D3-branes, which preserve the opposite supersymmetry. It is also obvious here that turning on both $G_{(2,1)}$ and $G_{(1,2)}$ breaks all space-time supersymmetry as would be expected by adding D3-branes and anti-D3-branes. This is entirely in agreement with the supergravity literature [34, 35], but such features would be difficult to see from the worldsheet in the RNS description since the space-time supersymmetry is not manifest.

3.4 F_1 and F_5 flux

It is straightforward to follow the above reasoning to deduce the form for the vertex operators corresponding to F_1 and F_5 fluxes where the F_5 is space-time filling and F_1 lies in the internal space. The result is that the vertex operators are given by

$$F_1 \rightarrow iF_1 \theta_L \theta_R \Omega_{(1,0)}, \quad (3.25)$$

$$F_5 \rightarrow F_5 \theta_L \theta_R \Omega_{(1,0)}. \quad (3.26)$$

For compact Calabi-Yau 3-folds with exactly $SU(3)$ holonomy and not a proper subgroup, the cohomology in dimension 1 vanishes: $h^{1,0} = h^{0,1} = 0$. Nevertheless, it is interesting to study these vertex operators for two reasons. Firstly, F_5 appears in conformally Calabi-Yau compactifications through a Bianchi identity, so it cannot be ignored in such backgrounds. Secondly, these flux operators appear in compactifications preserving 16 and 32 supersymmetries (e.g. compactifications on $M_6 = T^6$ preserve 32 supersymmetries). These extra supersymmetries are non-linearly realized in the hybrid formalism. The F_5 and F_1 strengths appear in multiplets which contain fields that are related to these non-linearly realized supersymmetries. Here we will describe them in the simplest possible case of compactification on T^6 .

The multiplets in question are semi-chiral on the worldsheet. They have conformal weights $(\frac{1}{2}, 0)$ and $(0, \frac{1}{2})$ and take the form

$$S_L = s_{iL}\psi_L^i, \quad \bar{S}_L = \bar{s}_{\bar{i}L}\bar{\psi}_L^{\bar{i}}, \quad S_R = s_{iR}\psi_R^i, \quad \bar{S}_R = \bar{s}_{\bar{i}R}\bar{\psi}_R^{\bar{i}}. \quad (3.27)$$

The physical state conditions for S_L are

$$G_{L,-\frac{1}{2}}^+ S_L = G_{L,\frac{1}{2}}^- S_L = G_{R,\frac{1}{2}}^+ S_L = G_{R,\frac{1}{2}}^- S_L = 0, \quad (3.28)$$

with analogous conditions for \bar{S}_L , S_R , and \bar{S}_R . In terms of the space-time fields s_{iL} these conditions are

$$\bar{\nabla}_{\dot{\alpha}L} s_{iL} = \nabla_{\dot{L}}^2 s_{iL} = \bar{\nabla}_R^2 s_{iL} = \nabla_R^2 s_{iL} = 0. \quad (3.29)$$

In the integrated vertex operator the semi-chiral space-time superfield admits a gauge transformation

$$\delta s_{iL} = \nabla_R^2 \Xi_{iL} + \bar{\nabla}_R^2 \Upsilon_{iL} \quad (3.30)$$

with semi-chiral parameter fields Ξ and Υ . These can be used to put s_{iL} into the Wess-Zumino gauge

$$s_{iL} = \theta_R \sigma^\mu \bar{\theta}_R B_{\mu i} + \theta_R \sigma_\mu \bar{\theta}_R \theta_L^\alpha (\psi_{\alpha i}^\mu + \sigma_{\alpha\dot{\alpha}}^\mu \chi_i^{\dot{\alpha}}) + \theta_L \theta_R \bar{\theta}_R^2 (iF_{5i} + F_{1i}) + \dots, \quad (3.31)$$

where we see that the F_1 and F_5 fluxes form a complex auxiliary scalar inside s_{iL} . A similar analysis holds for S_R , \bar{S}_L and \bar{S}_R . We will elucidate the role played by these auxiliary fields in section 4.2.

4. Physical effects and applications

In this section we study the consequences of turning on G_3 flux using the identifications made above. First, the supergravity description of the class of flux backgrounds we wish to study is reviewed. The same backgrounds are then studied from the string worldsheet and we demonstrate precise agreement with supergravity.

4.1 Conformally CY_3 backgrounds in supergravity

The class of flux solutions that we consider in this paper are well-studied in the supergravity literature. They are known as “conformally Calabi-Yau” solutions and are one of the simplest classes of flux solutions. Examples of this type were first constructed in [36], and a review with references is in [35]. We consider such backgrounds in order to compare our string calculations with known results in a regime where they should agree, that is, at large volume. Let us now review some of the relevant aspects of such compactifications and give a simple example we can study.

The geometry is warped, with metric given by

$$ds^2 = e^{2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu + e^{-2A(y)} g_{ij} dy^i dy^j, \quad (4.1)$$

where $e^{2A(y)}$ is the warp factor. The internal space is related to a Calabi-Yau 3-fold M_6 with metric g_{ij} by a conformal factor $e^{-2A(y)}$. There is three-form flux $G_3 = F_3 - \tau H_3$ which, in order to be compatible with $N = 1$ space-time supersymmetry, is taken to be a $(2, 1)$ -form:⁸ $G_3 \in H^{(2,1)}(M_6)$. The equations of motion are satisfied if the flux is primitive and imaginary self-dual. Additionally, there is a space-time filling 5-form flux which is proportional to derivatives of the warp-factor. The fluxes also need to satisfy the appropriate Bianchi-identities. The warp factor satisfies a Poisson equation sourced by the 3-form fluxes

$$\square_{CY} e^{4A} = \frac{1}{4} g_s^2 |G_3|^2 + \dots, \quad (4.2)$$

where the omitted terms are higher order in α' . These may correspond to D-branes or orientifold planes. In compact models, the presence of the orientifold planes is required in order for there to be non-trivial solutions to the Poisson equation (4.2). Further, one can generalize these solutions to include a holomorphically varying axio-dilaton τ which is sourced by the D7/O7-branes. For simplicity we will not consider any localized sources (either D3-branes, or 7-branes) and assume that $C_0 = 0$ and that the dilaton is constant and tunable.

In this paper, we restrict ourselves to studying flux perturbation theory. The string theory analysis is then simplified and we find that we can reproduce known results from supergravity. To this end, consider a Calabi-Yau 3-fold with non-compact $(2, 1)$ -cycles. Fluxes supported by these cycles are not quantized. In this situation one can turn on small amounts of G_3 flux and study the physical consequences of such a deformation order by order in the flux.

To simplify the comparison with most of the literature, we require our flux to be compatible with D3-branes (as opposed to anti-D3-branes). The corresponding supersymmetry conditions imply that the G_3 flux is of type $(2, 1)$. This condition is known to lift complex structure moduli and can be represented by a superpotential

$$W = \int G_3 \wedge \Omega. \quad (4.3)$$

in the low energy effective action [37, 38].

⁸Typically in the literature, the G_3 flux is taken to be of type $(2, 1)$ and imaginary self-dual. Space-time supersymmetry can also be achieved by choosing the flux to be $(1, 2)$ and imaginary anti-self-dual.

At second order in the flux, the space is deformed so as to satisfy the equations of motion: A non-trivial warp-factor satisfying equation (4.2) is generated. As we are doing perturbation theory in the flux, we can expand $e^{4A} = 1 + 4A + \dots$, with the omitted terms being of higher order. There is also the 5-form flux, which is required for the supergravity equations of motion to be satisfied. It takes the form $F_{0123i} = -4\partial_i A$. The Bianchi identities need to be satisfied, which for G_3 is trivial and for F_5 is equivalent to equation (4.2). We will now study such backgrounds from the string worldsheet and exactly re-derive these effects as a result of the deformations by the flux vertex operators.

4.2 Conformally CY_3 backgrounds in string theory

We now consider the class of backgrounds discussed in the previous section, that is, we turn on a small amount of G_3 flux along non-compactly supported cycles of a Calabi-Yau 3-fold. This corresponds to a deformation given by the integrated version

$$\begin{aligned} \mathcal{O}_{G_3} = & (G_L^-)_{-\frac{1}{2}}(G_R^-)_{-\frac{1}{2}} \left[(\theta_L + i\theta_R)^2 g_s \overline{G}_{(2,1)}^p \Omega_{(1,1)}^p \right] \\ & + (G_L^+)_{-\frac{1}{2}}(G_R^+)_{-\frac{1}{2}} \left[(\bar{\theta}_L - i\bar{\theta}_R)^2 g_s G_{(2,1)}^p \Omega_{(-1,-1)}^p \right]. \end{aligned} \quad (4.4)$$

of the vertex operator (3.24). We now check conformal invariance order by order in α' and find that there are corrections to the background at string tree level that are required to maintain the conformal invariance (see, for example, [39] for a similar discussion in context of RNS). These corrections will correspond to space-time warping.

Classical superconformal invariance. The classical conditions for superconformal invariance were discussed in section 3. There we found the following relevant features:

- The flux is required to be of type either (2, 1) or (1, 2). This is consistent with backgrounds compatible with either D3-branes or anti-D3-branes.
- One cannot add H_3 independently of the explicit B -field terms, which come from semi-chiral fields.

1-loop superconformal invariance and warping. The 1-loop β -function is well-known to be equivalent to the space-time fields obeying their equations of motion to lowest order in α' . The 1-loop β -function can be calculated by looking at the UV structure of the two-point function of two integrated vertex operators:

$$\left\langle \dots \int d^2z d^2w \mathcal{O}_{G_3}(z) \mathcal{O}_{G_3}(w) \right\rangle \sim \int d^2z d^2w \frac{\Pi_L^m \Pi_{mR} g_s^2 G_{(2,1)}^p \overline{G}_{(2,1)}^{\bar{p}} g_{a\bar{b}}}{|z-w|^4} + \dots, \quad (4.5)$$

where the ellipsis denotes arbitrary operator insertions and $g_{a\bar{b}}$ is the Zamolodchikov metric on the complex structure moduli space. The latter is defined by the 2-point function of Calabi-Yau chiral ring operators:

$$\langle \Omega_{(1,1)}^p(z, \bar{z}) \overline{\Omega}_{(-1,-1)}^{\bar{p}}(0, 0) \rangle = \frac{g^{p\bar{q}}}{|z|^4}. \quad (4.6)$$

The contribution (4.5) induces a divergence

$$-\log(\Lambda) \int d^2z \Pi_L^m \Pi_{mR} g_s^2 |G_{(2,1)}|^2 \tag{4.7}$$

which breaks conformal invariance. Conformal invariance can be preserved by introducing a correction to the background. Inspection of the component form of the physical states in section 2.4 reveals that the only available candidate that preserves 4d Poincare invariance is the real primary vertex operator in (2.23) with

$$U = (\theta_L \sigma^\mu \bar{\theta}_L) (\theta_R \sigma^\nu \bar{\theta}_R) \eta_{\mu\nu} A(y) + \dots, \tag{4.8}$$

where the scalar $A(y)$ is a function of the internal space. In the string gauge this component field becomes the trace of the metric.

One of the contributions of the operator (4.8) to the action is

$$\delta S_2 = \int d^2z \Pi_L^m \Pi_{mR} A(y) + \dots \tag{4.9}$$

When computing the contribution of the above term to the β -function one can use the covariant background field expansion, which gives a term

$$\int d^2z \Pi_L^m \Pi_{mR} y^i \bar{y}^{\bar{i}} \nabla_i \bar{\nabla}_{\bar{i}} A(y), \tag{4.10}$$

which, at 1-loop, gives a divergent term

$$\log(\Lambda) \int d^2z \Pi_L^m \Pi_{mR} \square_{CY} A. \tag{4.11}$$

The divergence is therefore canceled if the metric obeys the equation of motion

$$\square_{CY} A = g_s^2 |G_{(2,1)}|^2 + \dots, \tag{4.12}$$

where \square_{CY} is the Laplacian on the Calabi-Yau. We have thus succeeded in deriving the warping of space-time from the worldsheet and it agrees precisely with the equation (4.2) derived in supergravity to this order in the flux.⁹

We have not yet needed to use the space-time-filling 5-form. Based on the supergravity analysis in section 4.1, this should appear quadratically in the flux. It turns out that this space-filling 5-form flux appears in exactly the same way as the warping, only in a different superfield. The easiest way to see this is from the unintegrated $G_{(2,1)}$ flux vertex operator (3.24). Besides the divergence generating the warp factor, at 1-loop we get a divergent contribution from

$$\mathcal{V}_{G_{(2,1)}}(z) \mathcal{V}_{\bar{G}_{(2,1)}}(z) \sim \frac{g_s^2}{\epsilon^2} (\bar{\theta}_L - i\bar{\theta}_R)^2 (\theta_L + i\theta_R)^2 |G_{(2,1)}|^2 \tag{4.13}$$

giving rise to contributions with various θ dependence. One of them is just the one which gives the warp factor but there are others, for example, the $-2i\theta_L \theta_R \bar{\theta}_R^2$ term. This is going

⁹There are additional terms contained in the ellipsis from higher order calculations which give higher order corrections to the background.

to be canceled by the last term in the component expansion (3.31) for the semi-chiral field. To be more precise, if $S_L = i\theta_L\theta_R\bar{\theta}_R^2 F_{5i}\psi_L^i$, the contribution to the action has a term

$$(G_L^-)_{-\frac{1}{2}}(G_R^+)_{-\frac{1}{2}}(G_R^-)_{-\frac{1}{2}}S_L = \dots + i\theta_L\theta_R\bar{\theta}_R^2\bar{\nabla}_{\bar{j}}F_{5i}\partial_L y^i\partial_R\bar{y}^{\bar{j}} + \dots, \quad (4.14)$$

and, if we regularize the product $\partial_L y^i\partial_R\bar{y}^{\bar{j}}$ using $y^i(z)\bar{y}^{\bar{j}}(w) \rightarrow g^{i\bar{j}}\log(|z-w|^2 + \epsilon^2)$, we get

$$\langle\partial_L y^i\partial_R\bar{y}^{\bar{j}}\rangle = -\frac{g^{i\bar{j}}}{\epsilon^2}. \quad (4.15)$$

The divergence coming from the $G_{(2,1)}$ flux is canceled provided

$$\bar{\nabla}^i F_{5i} = 2g_s^2|G_{(2,1)}|^2, \quad (4.16)$$

and this gives the desired answer. All other terms coming from the operator product (4.13) cancel in a similar way.

We emphasize that what were separate conformal field theories for the $4d$ space-time and internal Calabi-Yau have become mixed into a single conformal field theory for the 10-dimensional target space as a whole. This is reflected in the warping of the $4d$ space-time and is expected to be a generic feature of flux backgrounds.

4.3 Superpotential for moduli

The presence of fluxes implies that a superpotential is generated for the Calabi-Yau complex structure moduli. In string theory this comes from a 3-point amplitude. In this section we derive this using the G_3 background discussed above. To this end, we briefly review how one defines string calculations in the hybrid formalism (see for example [22, 23, 15]) and then give the calculation for the superpotential.

One begins by twisting the superconformal algebra so that the operators in the 0-mode measure have weight 0. This twist is given by $(h_L, h_R) \rightarrow (h_L - \frac{q_L}{2}, h_R - \frac{q_R}{2})$ and introduces a +2-charge anomaly in the left- and right-moving sectors, which is to be canceled by the measure. Next, we define the measure on the internal space. By observing that

$$i\int_{CY_3}\Omega\wedge\bar{\Omega} = \frac{4}{3}\text{Vol}(CY_3), \quad (4.17)$$

we define the internal part of the measure to satisfy

$$\langle\Omega_L^+\Omega_R^-\rangle_{CY_3} = 1. \quad (4.18)$$

For the 4-dimensional space-time, momentum conservation is implemented as usual by integrating over d^4x . We also integrate over the zero modes of the θ coordinates. Finally, we must incorporate the chiral bosons. The final result is

$$\langle\theta_L^2\bar{\theta}_L^2\theta_R^2\bar{\theta}_R^2e^{-\rho_L-\rho_R}\Omega_L^+\Omega_R^-\rangle = 1. \quad (4.19)$$

In a Calabi-Yau background there are no chiral interactions or superpotential for the massless moduli (see for example [40]). However, in the conformally CY background discussed above, we find that there are scattering amplitudes which, in the low energy field

theory, are given by a superpotential. For example a scattering amplitude¹⁰

$$\langle \mathcal{V}^q \mathcal{V}^r \rangle_{S_0 + \delta S_{G_3}} = \langle \mathcal{V}_{G_{(2,1)}}^q \mathcal{V}^r \rangle_{S_0} \neq 0 \quad (4.20)$$

is now non-zero due to the flux background. Note that only vertex operators corresponding to complex structure moduli feel the presence of this flux. As computed in [15] this amplitude gives rise to a superpotential

$$\mathcal{W} = 3 \int d^4x d^2\theta^- g_s G_{(2,1)}^p \mathcal{V}^q(x, \theta^-) \mathcal{V}^r(x, \theta^-) C_{pqr}, \quad (4.21)$$

where C_{pqr} are $h^{2,1}$ intersection numbers, and we integrate over the unbroken space-time supersymmetry $d^2\theta^- = d\theta^- d\bar{\theta}^-$. Here, \mathcal{V}^p are the massive-moduli chiral superfields reduced to $N = 1$:

$$\mathcal{V}_{d=4} = \phi^p + \theta^- \chi^p + (\theta^-)^2 X_{--}^p + \dots, \quad (4.22)$$

where χ^p is the modulino corresponding to the complex structure modulus ϕ^p . Thus, we see that some of the complex structure moduli get masses depending on which fluxes are turned on. This is as expected from a supergravity analysis. At large volume, these amplitudes and superpotential are packaged into the usual supergravity superpotential¹¹

$$\mathcal{W} = \int G \wedge \delta\Omega. \quad (4.23)$$

We thus see that the hybrid easily calculates the quadratic contribution of the GVW superpotential at large volume.

5. Conclusion and future directions

In this paper we have studied type IIB string compactifications to 4 dimensions. We identified hybrid formalism vertex operators corresponding to NSNS and RR fluxes on the internal space. We showed that in flux perturbation theory one may easily compute various known physical quantities such as warping and the generation of a superpotential for the massive moduli. The manifest space-time supersymmetry automatically gives the supersymmetry conditions on the flux.

There are many interesting future directions to pursue. For example, in forthcoming work [41] we show that in the hybrid model the presence of RR flux implies that the space-time supersymmetry algebra develops a central charge. In the same work, we also show that RR flux naturally leads to non-anti-commutativity in space-time. This has been discussed previously [10, 11], and we find a natural analogue here. Finally, with an eye toward proving the existence of flux backgrounds, we have developed a formulation of the

¹⁰Normally such an amplitude would vanish by $SL(2, \mathbb{C})$ and supersymmetry.

¹¹Here $\delta\Omega$ means the variation of Ω induced by the presence of modulus fields considered when computing the amplitude. It should be remembered that vertex operators only contribute to *variations* of the complex and Kähler structures.

hybrid worldsheet theory in terms of RR ground states. Following [42], it can be argued that in the Born-Oppenheimer approximation flux vacua are stable against perturbative α' corrections in RG flow, giving evidence for the existence of flux vacua as string solutions.

There are other directions to pursue. In the context of flux compactifications one can study the analogue of the gauged linear sigma model, first developed in [43], with RR flux deformations [44]. One can also easily study D3-brane backgrounds in this formulation. One very interesting direction is to understand orientifolding, and thereby open the realm of compact solutions to analysis. Of course, the ultimate goal of this line of work is to develop methods of calculation to the point that one is able to study string solutions that are string scale. This would give insight into the properties of string vacua not attainable in supergravity and an understanding of the landscape of perturbative type II string vacua.

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